

**MULTIPLE POSITIVE SOLUTIONS OF SECOND-
ORDER STURM-LIOUVILLE BOUNDARY VALUE
PROBLEMS FOR IMPULSIVE
DIFFERENTIAL EQUATIONS**

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ABSTRACT. In this paper we study two types of impulsive Sturm-Liouville boundary value problems depending on the parameter λ . The existence of multiple positive solutions is obtained by applying a three critical points theorem given by Averna and Bonanno [2].

1. Introduction. In recent years, a great deal of work has been done on the study of the existence of solutions of boundary value problems for impulsive differential equations, by which the phenomena, such as many evolution processes, states changed at certain moments of time due to abrupt changes, are described. For relevant and recent references on impulsive differential equations, we refer to [13, 20–22, 26, 27]. For the background and applications of the theory of impulsive differential equations to different areas, we refer the reader to the monographs and some recent contributions as [7, 9, 11, 14, 18, 19, 29, 33, 34, 36, 37].

Some classical tools have been used to study impulsive differential equations in the literature. These classical tools include fixed point theorems in cones [1, 8, 12, 15] and the method of lower and upper solutions with monotone iterative technique, see [10, 16].

Critical point theory is a new method to deal with the existence of solutions for boundary value problems, please refer to [2–6, 17, 23–25,

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