

A NOTE ON THE PROJECTIVE VARIETIES OF ALMOST GENERAL TYPE

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ABSTRACT. A \mathbf{Q} -Cartier divisor D on a projective variety M is *almost nup*, if $(D, C) > 0$ for every very general curve C on M . An algebraic variety X is of *almost general type*, if there exists a projective variety M with only terminal singularities such that the canonical divisor K_M is almost nup and such that M is birationally equivalent to X . We prove that a complex algebraic variety is of almost general type if and only if it is neither uniruled nor covered by any family of varieties being birationally equivalent to minimal varieties with numerically trivial canonical divisors, under the minimal model conjecture. Furthermore we prove that, for a projective variety X with only terminal singularities, X is of almost general type if and only if the canonical divisor K_X is almost nup, under the minimal model conjecture.

1. Introduction. Throughout the paper every variety is defined over the field of complex numbers \mathbf{C} . We follow the notation and terminology in [8].

Definition 1.1. A \mathbf{Q} -Cartier divisor D on a projective variety X is *almost numerically positive* (*almost nup*, for short), if there exists a union F of at most countably many prime divisors on X such that $(D, C) > 0$ for every curve $C \not\subseteq F$ (i.e., if $(D, C) > 0$ for every very general curve C). We say that D is *quasi-numerically positive* (*quasi-nup*, for short), if D is nef and almost nup.

Definition 1.2. An algebraic variety X is *of almost general type*, if there exists a projective variety M with only terminal singularities such that the canonical divisor K_M is almost nup and that M is birationally equivalent to X .

Obviously, an algebraic variety X is of almost general type, if it is of general type (i.e., if the geometric Kodaira dimension $\kappa_{\text{geom}}(X)$ equals

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