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GRAPH DOUGLAS ALGEBRAS

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ABSTRACT. We develop a notion of Douglas algebras for the free semi-groupoid algebras arising from directed graphs. We analyze two extreme examples of the structure of such algebras, the first coming from the graphs with a single vertex, the second coming from cycle graphs. In the first example we demonstrate a lack of algebraic structure while in the second example we completely describe the Douglas algebras.

1. Introduction. The non self-adjoint operator algebras associated to directed graphs are often viewed as noncommutative generalizations of the algebra H^∞. In particular, there are many results which extend the classical results about H^∞ to the directed graph framework, including for example: a Beurling type theorem [**11**], a functional calculus [**10**] and interpolation results [**8**].

This paper is a general discussion of the notion of Douglas algebras for directed graph operator algebras. It takes its shape, primarily, as a presentation of two classes of examples at opposite extremes of results we might hope for from the commutative context. The first class of examples consists of the algebras $\mathcal{L}_n$ arising from the graph with a single vertex and n directed edges. For the second class of examples we discuss the cycle graphs.

The starting points for this article are a pair of results on the spaces of the form $H^\infty + C(\mathbf{T})$ in the context of directed graphs. The first comes from [**4**] where in a discussion after Lemma 1.11 an analogue of the space $H^\infty + C(\mathbf{T})$ in the context of $\mathcal{L}_n$ is shown to be closed, although it is not an algebra. The second is from a paper [**1**] where a slightly different analogue (a distinction we will take up in Section 3) was shown to be a closed algebra in the case that the graph is a cycle graph of length n.

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