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DIRECT SUMS OF VORONOVSKAJA'S TYPE FORMULAS

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ABSTRACT. Using a general procedure we consider some combination of different approximation processes by means of projections on orthogonal subspaces. We concentrate our attention on some particular positive approximation processes in spaces of L^2-real functions in order to satisfy a prescribed Voronovskaja's type formula.

1. Direct sums of approximation processes and associated Voronovskaja's type formulas. We are mainly interested in the application of a general and simple method which consists in constructing a new approximation process starting with a decomposition of a Hilbert space into the direct sum of orthogonal subspaces and associating to each subspace an assigned approximation process.

In this way we obtain some noteworthy results regarding the possibility of obtaining new Voronovskaja's type formulas from assigned ones and extending the class of differential problems under consideration.

The general method can actually be applied in different settings. Indeed, we may have the necessity of using different approximation processes on orthogonal subspaces as done in Section 2 in connection with Bernstein-Kantorovich and Bernstein-Durrmeyer operators; this may happen for example in studying diffusion models in population genetics where different factors may depend on the subspace containing the initial condition. Indeed, it is well known that the differential operator arising from the Voronovskaja formula for both Bernstein-Kantorovich and Bernstein-Durrmeyer operators describes the evolution process associated with some diffusion models in population genetics through the representation given in (2.13) which depends only on the initial condition u_0 in (2.14). Hence, the method used in Section 2 allows

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