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STABILITY CLASSES OF SECOND-ORDER LINEAR RECURRENCES MODULO 2^k II

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ABSTRACT. We classify the 2^k-blocks of second-order recurrence sequences with parameter $b \equiv 5 \pmod 8$ and identify stability classes modulo 2.

1. Introduction. Let $w(a,b)$ denote the second-order recursive sequence (w_i) determined by integer parameters a and b, initial integer terms w_0 and w_1, and recursion

$$w_i = aw_{i-1} + bw_{i-2}. \tag{1.1}$$

For each integer m, let $(\overline{w}_i)$ denote the corresponding sequence of residues modulo m. The residue sequence $(\overline{w}_i)$ is periodic, and purely periodic when m is relatively prime to b. Despite their simple definition, such sequences remain a source of many interesting open questions, among them the determination of the period, restricted period, and residue frequency distribution.

Considerable progress has been made in understanding the frequency distributions of sequences $(\overline{w}_i)$ when the modulus m is a prime power, much of it motivated by pioneering work of Eliot Jacobson beginning with [**10**]. Suppose that $m = p^k$ is a power of a prime p. Let $\lambda_k = \lambda_w(p^k)$ be the (least) period of $(\overline{w}_i)$ and, for each residue d modulo p^k, $\nu_w(d,p^k)$ the number of times that the residue d appears in a single cycle of the recurrence $(\overline{w}_i)$. Let $\Omega_w(p^k)$ be the image of the frequency distribution function $\nu_w(d,p^k)$, i.e.,

$$\Omega_w(p^k) = \{\nu_w(d,p^k) \mid d \in \mathbf{Z}\}.$$

In [**10**], Jacobson observed that when $w(a,b)$ is the Fibonacci sequence, the sets $\Omega_w(2^k)$ are eventually constant as a function of k, and hence

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