

## GENERALIZED BI-CIRCULAR PROJECTIONS ON $\mathcal{C}(\Omega, X)$

FERNANDA BOTELHO AND JAMES E. JAMISON

ABSTRACT. Let  $\Omega$  be a connected compact Hausdorff space and  $X$  a Banach space for which the Strong Banach-Stone property is valid. We give a complete characterization of the generalized bi-circular projections on Banach spaces of vector valued continuous functions. We also observe that generalized bi-circular projections on  $\mathcal{C}(\Omega, X)$  are bi-contractive.

**1. Introduction.** Let  $(X, \|\cdot\|)$  be a complex Banach space, and let  $P : X \rightarrow X$  be a linear projection. A basic problem in Banach space theory is to determine the structure of the projections on a given Banach space and provide characterizations of their ranges. The existence of Hermitian projections on a Banach space and its connection with the geometric properties of the underlying space was investigated by Berkson, in [2]. Contractive and bi-contractive projections on  $L_p$  spaces and on spaces of continuous functions, as well as circular projections in a variety of settings, are among the standard problems addressed in the literature, see for example [3, 6, 16]. Recently, a new class of projections, namely bi-circular projections, was proposed and has been a focus of research interest, see [17]. A projection is called bi-circular if  $e^{i\alpha}P + e^{i\beta}(I - P)$  is an isometry, for all real numbers  $\alpha$  and  $\beta$ . These projections were studied in many different settings by Stacho and Zalar, see [18]. Furthermore, it was shown in [9] that these projections are norm Hermitian. As a consequence, many results on bi-circular projections follow from previously established results on Hermitian operators on Banach spaces. Fosner, Ilisevic and Li, in [6], have introduced a generalization of bi-circular projections by requiring  $P + \lambda(I - P)$  to be an isometry, for some modulus 1 complex number  $\lambda \neq 1$ . They obtained interesting characterizations of these projections in the finite-dimensional case for both real and complex vector spaces. It is of interest to characterize these projections for other Banach spaces.

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