

ASYMPTOTIC INTEGRATION UNDER WEAK DICHOTOMIES

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ABSTRACT. In a classical result, Levinson considered perturbations of diagonal systems of differential equations. He showed that the unperturbed and the perturbed system are strongly asymptotically equivalent if the entries of the diagonal matrix satisfy a certain dichotomy condition and if the perturbation is absolutely integrable. Here we are interested in diagonal linear systems that satisfy dichotomy conditions which are weaker than Levinson's. We show that the diagonal system is still strongly asymptotically equivalent to a perturbed system provided that the perturbation is sufficiently small. We also generalize these results to perturbations of Jordan matrices. We give some corresponding results for perturbations of systems of difference equations and conclude with examples.

1. Introduction. In [3], we introduced a concept called *strong asymptotic equivalence* between two linear systems $x' = A(t)x$ and $y' = B(t)y$ defined for $t \geq t_0$. This means that there exist corresponding fundamental solutions related by the asymptotic equation

$$X(t) = [I + o(1)] Y(t) \text{ as } t \rightarrow +\infty.$$

While it can be shown (see Theorem 5) that any two systems are strongly asymptotically equivalent provided $A(t) - B(t)$ is "sufficiently small," this general result does not yield a practical (close to optimal) bound. Instead, in most applications, one should think of $A(t)$ as having a certain structure (e.g. diagonal, block-diagonal, Jordan) and $B(t) = A(t) + R(t)$ as a perturbed system. Then a simpler problem is, given the structure of A and certain easily-checked properties, to determine suitable smallness conditions on R which imply strong asymptotic equivalence to the unperturbed system.

Keywords and phrases. Differential equations, perturbations, dichotomy conditions, asymptotic behavior, difference equations.

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