ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 40, Number 1, 2010

SOME REMARKS ON SPECIAL SUBORDINATORS

RENMING SONG AND ZORAN VONDRAČEK

ABSTRACT. A subordinator is called *special* if the restriction of its potential measure to $(0,\infty)$ has a decreasing density with respect to the Lebesgue measure. In this note we investigate what type of measures μ on $(0,\infty)$ can arise as Lévy measures of special subordinators and what type of functions $u:(0,\infty)\to[0,\infty)$ can arise as potential densities of special subordinators. As an application of the main result, we give examples of potential densities of subordinators which are constant to the right of a positive number.

1. Introduction. A function $\phi:(0,\infty)\to(0,\infty)$ is called a Bernstein function if it admits a representation

$$\phi(\lambda)=a+b\lambda+\int_0^\infty (1-e^{-\lambda x})\,\mu(dx), \tag{1.1}$$

where $a\ge 0$ is the *killing term*, $b\ge 0$ the *drift* and μ a measure on $(0,\infty)$ satisfying $\int_0^\infty (x\wedge 1)\,\mu(dx)<\infty$, called the *Lévy measure*. By defining $\mu(\{\infty\})=a$, the measure μ is extended to a measure on $(0,\infty]$. The function $\overline{\mu}(x):=\mu((x,\infty])$ on $(0,\infty)$ is called the *tail* of the Lévy measure. Using integration by parts, formula (1.1) becomes

$$\phi(\lambda)=b\lambda+\lambda\int_0^\infty e^{-\lambda x}\bar{\mu}(x)\,dx. \tag{1.2}$$

The function ϕ is called a special Bernstein function if the function $\psi:(0,\infty)\to(0,\infty)$ defined by $\psi(\lambda):=\lambda/\phi(\lambda)$ is again a Bernstein function. Let

$$\psi(\lambda)=\tilde{a}+\tilde{b}\lambda+\int_0^\infty (1-e^{-\lambda x})\,\nu(dx) \tag{1.3}$$

2010 AMS *Mathematics subject classification.* Primary 60G51, Secondary 60J45, 60J75.
Keywords and phrases. Subordinator, potential density, Lévy measure, Bernstein function, log-convex function.
The research of the first author is supported in part by a joint US-Croatia grant INT 0302167. The research of the second author is supported in part by MZOS grant 037-0372790-2801 of the Republic of Croatia.
Received by the editors on March 14, 2007, and in revised form on August 2, 2007.
DOI:10.1216/RMJ-2010-40-1-321