ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 40, Number 1, 2010

LEGENDRE SURFACES WITH HARMONIC MEAN CURVATURE VECTOR FIELD IN THE UNIT 5-SPHERE

TORU SASAHARA

ABSTRACT. We obtain the explicit representation of Legendre surfaces in the unit 5-sphere with harmonic mean curvature vector field, under the condition that the mean curvature function is constant along a certain special direction.

1. Introduction. It is well known that an odd-dimensional unit sphere S^{2n+1} is equipped with the standard Sasakian structure (g, ϕ, η, ξ), see Section 2. The study of minimal Legendre submanifolds in S^{2n+1} is a very active field and closely related to one of minimal Lagrangian submanifolds in complex projective space.

A natural generalization of a minimal submanifold is a submanifold with parallel mean curvature vector field. However, Legendre submanifolds in S^{2n+1} with parallel mean curvature vector field are in fact minimal, see [**9**]. So, in case the ambient space is S^{2n+1}, as a generalization of minimal Legendre submanifolds, it is natural to consider Legendre submanifolds whose mean curvature vector field H is harmonic with respect to the normal Laplacian Δ^D, that is,

$$\Delta^D H = 0. \tag{1.1}$$

The purpose of this paper is to study the class of nonminimal Legendre surfaces satisfying (1.1) in the unit 5-sphere. On nonminimal Legendre submanifolds, there exists a special vector field: ϕH. Moreover, in case the dimension is 2, up to signs, there is a unique unit vector field normal to ϕH. We denote it by $(\phi H)^{\perp}$. In this paper, under the condition that the square of the mean curvature function is constant along one of ϕH and $(\phi H)^{\perp}$, we completely determine nonminimal Legendre surfaces satisfying (1.1) in the unit 5-sphere.

2010 AMS *Mathematics subject classification.* Primary 53C42, Secondary 53B25.
Received by the editors on December 20, 2005, and in revised form on September 12, 2007.
DOI:10.1216/RMJ-2010-40-1-313