

SOME PROPERTIES OF BASE-MATROIDS OF ARBITRARY CARDINALITY

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ABSTRACT. On a set of arbitrary cardinality, this paper introduces a new construction which is called the base-matroid of arbitrary cardinality. After presenting the base axiom system for a matroid of arbitrary cardinality, it discusses the new construction with the base axiom system and obtains that the new construction is a matroid of arbitrary cardinality. Afterwards, with the assistance of both lattice and matroid theories, it discusses the lattice construction of closed saturated sets relative to a simple matroid of arbitrary cardinality.

1. Introduction and preliminaries. In this paper we will use the techniques of finite base-matroid for reference to produce a base-matroid of arbitrary cardinality. Initially, as a test for what can be achieved by using this approach, we will study the base axioms for a base-matroid of arbitrary cardinality. Welsh in [9] and Novetti and White in [7] have identified the base axioms for a finite matroid. This paper, however, is the first to show the base axioms for a matroid of arbitrary cardinality.

Mao in [5] presents a method to consider the relationship between a geometric lattice and the family of closed sets of a simple matroid of arbitrary-cardinality. Additionally, [4] discusses the construction of a Boolean lattice of closed saturated sets relative to a simple matroid of arbitrary cardinality. Combining the ideas of [4, 5] this paper also shows some results about the base-matroid of arbitrary cardinality for a simple matroid of arbitrary cardinality.

We will begin by reviewing and presenting the knowledge needed to continue. In what follows, E is assumed to be some arbitrary-possibly infinite-set. For $X \subseteq E$, $|X|$ denotes the cardinality of X .

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