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BANACH-STEINHAUS TYPE THEOREMS FOR STATISTICAL AND $\mathcal{I}$-CONVERGENCE WITH APPLICATIONS TO MATRIX MAPS

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ABSTRACT. Let (A_n) be a sequence of bounded linear operators from a Banach space X into a Banach space Y. It is proved that if X has a countable fundamental set Φ and the ideal $\mathcal{I}$ of subsets of $\mathbf{N}$ has property (APO), then $(A_n x)$ is boundedly $\mathcal{I}$-convergent for each $x \in X$ if and only if $\sup_n \|A_n\| < \infty$ and $(A_n \phi)$ is $\mathcal{I}$-convergent for any $\phi \in \Phi$. This result is applied to characterize some sequence-to-sequence transformations defined by infinite matrices of bounded linear operators.

1. Introduction and auxiliary results. Let $\mathbf{N} = \{1, 2, \dots\}$, and let X, Y be two Banach spaces over the field $\mathbf{K}$ of real or complex numbers. A subset Φ of X is called *fundamental* if the linear span of Φ is dense in X. By $B(X,Y)$ we denote the space of all bounded linear operators from X into Y. We write $\sup_n$, $\lim_n$, $\sum_n$, $\cup_n$ and $\cap_n$ instead of $\sup_{n\in\mathbf{N}}$, $\lim_{n\to\infty}$, $\sum_{n=1}^{\infty}$, $\cup_{n=1}^{\infty}$ and $\cap_{n=1}^{\infty}$, respectively.

Let $A_n \in B(X,Y)$, $n \in \mathbf{N}$. A well-known *principle of uniform boundedness* asserts that if $\sup_n \|A_n x\| < \infty$ for every $x \in X$, then there exists a constant $M > 0$ such that

$$\|A_n\| \le M, \quad n \in \mathbf{N}. \tag{1.1}$$

By investigation of the convergence of various linear processes the following corollary from this principle is useful (see, for example, [**4**, page 248] or [**9**, page 173]).

Theorem 1 (Banach-Steinhaus). *Let $\Phi \subset X$ be a fundamental set. The limit $\lim_n A_n x$ exists for any $x \in X$ if and only if* (1.1) *holds and $\lim_n A_n \phi$ exists for every $\phi \in \Phi$. Moreover, the limit operator*

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