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REGULAR FUNCTIONS ON THE SPACE OF CAYLEY NUMBERS

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ABSTRACT. In this paper we present a new definition of regularity on the space $\mathbf{O}$ of Cayley numbers (often referred to as octonions), based on a Gateaux-like notion of derivative. We study the main properties of regular functions, and we develop the basic elements of a function theory on $\mathbf{O}$. Particular attention is given to the structure of the zero sets of such functions.

1. Introduction. Let $\mathbf{O}$ denote the nonassociative, alternative, division algebra of real Cayley numbers (also known as octonions). We refer the reader to the excellent survey [**2**] for a thorough discussion of the importance and interest of this object. A simple way to describe the construction of this algebra is to consider a basis $\mathcal{E} = \{e_0 = 1, e_1, \dots, e_6, e_7\}$ of $\mathbf{R}^8$ and relations

$$e_\alpha e_\beta = -\delta_{\alpha\beta} + \psi_{\alpha\beta\gamma} e_\gamma, \quad \alpha, \beta, \gamma = 1, 2, \dots, 7,$$

where $\delta_{\alpha\beta}$ is the Kronecker delta, and $\psi_{\alpha\beta\gamma}$ is totally antisymmetric in α, β, γ, nonzero and equal to 1 on the seven combinations in the following set

$$\sigma = \{(1,2,3), (1,4,5), (2,4,6), (3,4,7), (2,5,7), (1,6,7), (5,3,6)\},$$

so that every element in $\mathbf{O}$ can be written as $w = x_0 + \sum_{k=1}^{7} x_k e_k$. One can then define in a natural fashion its conjugate $\overline{w} = x_0 - \sum_{k=1}^{7} x_k e_k$, and its square norm $|w|^2 = w\overline{w} = \sum_{k=0}^{7} x_k^2$.

The basic elements of $\mathbf{O}$ can be written (see e.g. [**16**]) as

$$e_0 = 1, e_1, e_2, e_1e_2, e_4, e_1e_4, e_2e_4, (e_1e_2)e_4.$$

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