

ON THE REACHABILITY AND CONTROLLABILITY
OF POSITIVE LINEAR TIME-INVARIANT
DYNAMIC SYSTEMS WITH INTERNAL AND
EXTERNAL INCOMMENSURATE POINT DELAYS

M. DE LA SEN

ABSTRACT. This paper is devoted to the characterization of reachability and controllability properties of singular linear time-delay, time-invariant dynamic positive systems subject to constant known internal and external point delays. The research is performed on two different levels, namely, based on a general nonunique solution trajectory including discontinuities at points of the initialization time-interval and based on a particular solution with initial conditions in an appropriate subspace of the whole potential admissible set which avoids such discontinuities and guarantees uniqueness. In this second case, the positivity, reachability and controllability characterizations of the Weierstrass canonical state-space realization are also investigated.

I. Introduction. This paper investigates the general state-trajectory solution and the reachability and controllability properties of singular linear time-delay, time-invariant dynamic positive systems subject to a finite number of constant known internal and external point delays. The general problem statement is concerned with the possible presence of impulses at zero of the initial conditions in the regular impulsive case which results in the loss of uniqueness of the state-trajectory solution, [11]. A certain subset of initial conditions of the fast dynamics partial state guarantees the state-trajectory solution in both the natural state variables of the given problem and its counterpart in the Weierstrass canonical form. The state-trajectory of the regular (referred to as well as solvable [1, 13, 15, 19]) state impulse-free case where the nilpotent matrix of the general singular system becomes zero is also obtained

2010 AMS *Mathematics subject classification.* Primary 93C23, 34K45, 34L40, 93C05, 93C23.

Keywords and phrases. Controllability, positive systems, reachability, singular systems, time-delay systems.

Partial support of this work provided by the Spanish Ministry of Education through Grant DPI 2006-00714.

Received by the editors on July 12, 2007 and accepted on September 28, 2007.

DOI:10.1216/RMJ-2010-40-1-177 Copyright ©2010 Rocky Mountain Mathematics Consortium