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EXPLICIT ESTIMATE ON PRIMES BETWEEN CONSECUTIVE CUBES

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ABSTRACT. We give an explicit form of Ingham's theorem on primes in the short intervals and show that there is at least one prime between every two consecutive cubes x^3 and $(x+1)^3$ if $\log\log x \geq 15$.

1. Introduction. Studies about certain problems in number theory are often connected to those about the distribution of prime numbers; problems about the distribution of primes are among the central ones in number theory. One problem concerning the distribution of primes is the distribution of primes in certain intervals. For example, Bertrand's postulate asserts that there is a number B such that, for every $x > 1$, there is at least one prime number between x and Bx. If the interval $[x, Bx]$ is replaced by a "short interval" $[x, x + x^\theta]$, then the problem is more difficult.

In 1930, Hoheisel showed that there is at least one prime in the above mentioned "short interval" with $\theta = 1-(1/33,000)$ for sufficiently large x's, see [**13**]. Ingham [**15**], in 1941, proved that there is at least one prime in $[x, x+x^{3/5+\varepsilon}]$, where ε is an arbitrary positive number tending to zero whenever x is tending to infinity, for "sufficiently large" x's. This implies that there is at least one prime between two consecutive cubes if the numbers involved are "large enough." One of the better results in this direction, conjectured by using the Riemann hypothesis, is that there is at least one prime between $[x, x+x^{1/2+\varepsilon}]$ for "sufficiently large" x's. The latter has not been proved or disproved, though better results than Hoheisel's and Ingham's are available. For example, one may see [**2, 3, 12, 15, 17, 18, 19, 26, 28**].

These kinds of results would have many useful applications if they were "explicit" (with all constants being determined explicitly). For references in other directions with explicit results, one can see [**4,**

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