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GLEASON-KAHANE-ZELASKO TYPE THEOREMS FOR COMPLEX RIESZ ALGEBRAS

YOUSSEF AZOUZI AND KARIM BOULABIAR

Dedicated to Professor Melvin 'Mel' Henriksen
on the occasion of his 80th birthday

ABSTRACT. Let $\mathfrak{A}$ be a complex f-algebra with a unit element e. It is shown that a linear functional f on $\mathfrak{A}$ is a lattice homomorphism with $f(e) = 1$ if and only if $f(\mathfrak{a}) \in \sigma(\mathfrak{a})$ for all $\mathfrak{a} \in \mathfrak{A}$. More generally, let $\mathfrak{A}$ be a complex Riesz algebra with a positive unit element e. It turns out that the principal band $\mathfrak{B}_e$ in $\mathfrak{A}$ generated by e is a projection band in $\mathfrak{A}$. Moreover, a linear functional f on $\mathfrak{A}$ is a lattice homomorphism with $f(e) = 1$ if and only if $f(\mathfrak{a}) \in \sigma(P_e(\mathfrak{a}))$ for all $\mathfrak{a} \in \mathfrak{A}$, where P_e denotes the band projection of $\mathfrak{A}$ onto $\mathfrak{B}_e$. It follows that if E is a Dedekind complete complex Riesz space then a linear functional f on $L^r(E)$ is an identity preserving lattice homomorphism if and only if for each $T \in L^r(E)$ the scalar $f(T)$ is a spectral value in $L(E)$ of the diagonal component $D(T)$ of T.

1. Introduction.

At the end of the 1960s, Zelasko [**20**] proved one of the most famous characterizations of a complex-valued algebra homomorphism on a complex Banach algebra $\mathfrak{A}$ with a unit element. Namely, a nonzero linear functional f on $\mathfrak{A}$ is an algebra homomorphism on $\mathfrak{A}$ if and only if $f(\mathfrak{a}) \in \sigma(\mathfrak{a})$ for all $\mathfrak{a} \in \mathfrak{A}$, where $\sigma(\mathfrak{a})$ denotes the spectrum of $\mathfrak{a}$ in $\mathfrak{A}$. The commutative version of this remarkable result was obtained earlier by Gleason [**6**] and, independently, by Kahane and Zelasko [**14**]. Henceforth, this result is known as the Gleason-Kahane-Zelasko theorem in the vast literature on the subject. In this regard, Jarosz [**13**] gave an interesting historical account which can be consulted for more bibliographic information concerning the Gleason-Kahane-Zelasko theorem. It is well known that, to a quite large extent,

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