ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 39, Number 6, 2009

A DIRECT PROOF OF A THEOREM ON MONOTONICALLY NORMAL SPACES FOR GO-SPACES

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ABSTRACT. Let X be a GO-space. We give a direct proof of the following fact: If $\mathcal{O}$ is an open cover of X, then $\mathcal{O}$ has a σ-disjoint open, partial refinement $\mathcal{V}$ such that $X\setminus\cup\mathcal{V}$ is the union of a discrete family of stationary subsets of regular uncountable cardinals.

1. Introduction and terminology. We say that an open cover $\mathcal{O}$ of a topological space X has the $(\star)$ *property*, if $\mathcal{O}$ has a σ-disjoint open partial refinement $\mathcal{V}$ such that $X\setminus\cup\mathcal{V}$ is the union of a discrete family of closed subspaces which are homeomorphic to some stationary subset of a regular uncountable cardinal. A topological space X is said to have the $(\star)$*property*, if each open cover $\mathcal{O}$ of X has the $(\star)$ property.

In [**1**], Balogh and Rudin have proved that monotonically normal spaces have the $(\star)$ property, and in [**3**] it was proved that any GO-space is monotonically normal. So, GO-spaces have the $(\star)$ property. In this paper we give a direct and simple proof of this fact using "the method of coherent collections" described in [**4**].

A subset C of a linearly ordered set $(X,\leq)$ is called *convex* if

$$\{x\in X : a\leq x\leq b\}\subseteq C$$

for each $a,b\in C$ with $a\leq b$. Let $(X,\mathcal{T})$ be a topological space and $\leq$ a linear order on X. Recall that $(X,\mathcal{T},\leq)$ is called a *generalized ordered space* (or GO-*space*), if $\mathcal{T}$ contains usual open interval topology on X and has a base consisting of convex subsets.

Let $\mathcal{B}$ be a collection of subsets of X and $A\subseteq X$. Then we write

$$st(A,\mathcal{B})=\bigcup\{B\in\mathcal{B} : A\cap B\neq\varnothing\}.$$

2000 AMS *Mathematics subject classification.* Primary 54F05, 54D20.
Keywords and phrases. GO-space, refinement, stationary set.
Received by the editors on June 6, 2004, and in revised form on May 13, 2007.
DOI:10.1216/RMJ-2009-39-6-2067