

COMPOSITION OPERATORS BETWEEN
BLOCH-TYPE SPACES
AND MÖBIUS INVARIANT Q_K SPACES

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ABSTRACT. A characterization of the boundedness and compactness of a composition operator $C_\phi f = f \circ \phi$ acting from the Bloch type spaces \mathcal{B}^α to the Möbius invariant spaces Q_K is given. In particular, estimates for the essential norm of such an operator are obtained.

1. Introduction and main results. Let $\phi : \mathbf{D} \rightarrow \mathbf{D}$ be an analytic map of the unit disc $\mathbf{D} = \{z : |z| < 1\}$ into itself. The map ϕ induces a linear composition operator $C_\phi f = f \circ \phi$ on space $\mathcal{H}(\mathbf{D})$ of all analytic functions on the unit disc. A fundamental problem in the study of composition operators is to characterize in terms of the function theoretic properties of ϕ , the boundedness and compactness of restrictions of C_ϕ to various Banach spaces of analytic functions.

Recall that a bounded linear map T from a Banach space X into a Banach space Y is called *compact* (weakly compact) if it maps the closed unit ball of X onto a relatively compact (a relatively weakly compact) set in Y . The *essential norm* of T is defined to be the distance to the compact operators, that is,

$$\|T\|_e = \inf\{\|T - S\| : S \text{ is compact}\}.$$

Since $\|T\|_e = 0$ if and only if T is compact, estimates for $\|T\|_e$ give conditions for T to be compact.

For $s > -1$, consider the weighted Dirichlet space D_s of all analytic functions on the unit disc \mathbf{D} for which

$$\|f\|_{D_s}^2 = \int_{\mathbf{D}} |f'(z)|^2 (1 - |z|^2)^s dA(z) < \infty.$$

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