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CHARACTERIZATION OF STRICT CONVEXITY FOR LOCALLY LIPSCHITZ FUNCTIONS

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ABSTRACT. The first goal of this paper is improvement of our previous result (Nonlinear Anal. TMA **57** (2004), 85–97), i.e., the characterization of convexity for regularly locally Lipschitz functions by means of the second-order upper (Dini) directional derivative.

Using the (Dini) type of generalized second-order directional derivative, we also provide the characterization of strict convexity for locally Lipschitz functions. As application of this characterization, we can obtain the second-order sufficient optimality condition introduced by Cominetti and Correa.

1. Introduction and preliminaries. Strict convexity plays a very important role in mathematics. For example, it is a well-known fact that it suffices to solve the inclusion $0 \in \partial f(x)$ to find a strict local minimum of the strict convex real function$f$ defined on an open subset of X, where $\partial f(x)$ denotes the convex subdifferential of f at x [**26**] and the symbol X is reserved for a real Banach space with the norm $\|\cdot\|$ in this paper.

We can give the following characterization of strict convexity by means of classical second-order directional derivatives. This result is proved, e.g., in [**7**] for $X = \mathbf{R}$, but we note that the dimension of X is not important in this case. By S_X, we will mean the set $\{h \in X : \|h\| = 1\}$. For $x \in X$, $(u, v) \in X^2$, we denote

$$f''(x; u, v) = \lim_{t \to 0} \frac{\langle \bigtriangledown f(x + tu) - \bigtriangledown f(x), v \rangle}{t},$$

where $\bigtriangledown f(x)$ is the symbol for the Gâteaux derivative of f at x.

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