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CAUCHY-RASSIAS STABILITY OF SESQUILINEAR n-QUADRATIC MAPPINGS IN BANACH MODULES

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ABSTRACT. We prove the Cauchy-Rassias stability of a sesquilinear n-quadratic mapping in a left Banach module over a unital C^*-algebra.

1. Introduction. Let X and Y be Banach spaces. Consider $f : X \to Y$ to be a mapping such that $f(tx)$ is continuous in $t \in \mathbf{R}$ for each fixed $x \in X$. Rassias [**11**] introduced the following inequality that we call *Cauchy-Rassias inequality*: Assume that there exist constants $\theta \geq 0$ and $p \in [0, 1)$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \theta(||x||^p + ||y||^p)$$

for all $x, y \in X$. Rassias [**11**] showed that there exists a unique $\mathbf{R}$-linear mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \leq \frac{2\theta}{2 - 2^p} ||x||^p$$

for all $x \in X$. The above inequality has provided a lot of influence in mathematical analysis in the development of what we now call *Hyers-Ulam-Rassias stability* of functional equations.

The norm on an inner product space satisfies the classical parallelogram equality

$$\|x + y\|^2 + \|x - y\|^2 = 2\|x\|^2 + 2\|y\|^2.$$

The functional equation

$$f(x + y) + f(x - y) = 2f(x) + 2f(y)$$

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