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THE VARIATIONAL MCSHANE INTEGRAL IN LOCALLY CONVEX SPACES

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ABSTRACT. The variational McShane integral for functions taking values in a locally convex space is defined, and it is characterized by means of the p-variations of the indefinite Pettis integral.

1. Introduction. Riemann generalized integrals taking values in locally convex space have been studied in [**9, 10**]. In this paper we go a bit further in studying the variational McShane integral for functions defined in a σ-finite quasi Radon measure space and taking values in locally convex spaces. In [**9**] it is proved that if the domain is a compact subinterval of the real line, the family of McShane integrable functions coincide with that of variationally McShane integrable ones if and only if the space is nuclear. It is known that, for Banach valued functions, the family of variationally McShane integrable functions can be significantly larger than that of Bochner integrable ones [**2**]. We extend this result to the setting of locally convex spaces. We prove some properties of the variational McShane integral. The main result is the characterization of the family of variational McShane integrable functions by means of the Pettis integrability and of the fact that, for each semi-norm p, the p-variation of the indefinite Pettis integral is moderated, Theorem 4. The proof is based on differentiability of the primitive with respect to a suitable base which we introduce in a quasi Radon measure space, Theorem 3. As a corollary we get that in compact Radon measure spaces the family of variationally McShane integrable functions coincide with that of integrable by semi-norm ones, Theorem 6. Moreover, we give an example of a function which is measurable by semi-norm and Pettis integrable, but such that the p-variation of the indefinite Pettis integral is not moderated.

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