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GENERALIZED BASKAKOV-BETA OPERATORS

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ABSTRACT. Very recently Wang [**9**] introduced the modified form of Baskakov-beta operators and obtained a Voronovskaja type asymptotic formula for these operators. We extend the study and here we estimate a direct result in terms of higher order modulus of continuity and an inverse theorem in simultaneous approximation for these new modified Baskakov-beta operators.

1. Introduction. For $f \in C_\gamma[0,\infty) \equiv \{f \in C[0,\infty) : f(t) = O(t^\gamma)$ as $t \to \infty$ for some $\gamma > 0\}$ and $\alpha > 0$, Wang [**9**] introduced modified Baskakov-beta operators as

(1)
$$B_{n,\alpha}(f,x) = \sum_{k=0}^{\infty} p_{n,k,\alpha}(x) \int_0^\infty b_{n,k,\alpha}(t) f(t)\,dt = \int_0^\infty W_{n,\alpha}(x,t) f(t)\,dt$$

where

$$p_{n,k,\alpha}(x) = \frac{\Gamma(n/\alpha + k)}{\Gamma(k+1)\Gamma(n/\alpha)} . \frac{(\alpha x)^k}{(1+\alpha x)^{(n/\alpha)+k}},$$
$$b_{n,k,\alpha}(t) = \frac{1}{B(n/\alpha, k+1)} \frac{\alpha(\alpha t)^k}{(1+\alpha t)^{n/\alpha+k+1}}$$

and

$$W_{n,\alpha}(x,t) = \sum_{k=0}^{\infty} p_{n,k,\alpha}(x) b_{n,k,\alpha}(t).$$

The norm- $||.||_\gamma$ on the class $C_\gamma[0,\infty)$ is defined as $||f||_\gamma = \sup_{0<t<\infty} |f(t)|t^{-\gamma}$.

As a special case $\alpha = 1$, the operators defined by (1) reduce to the well known Baskakov-beta operators [**5**]. Wang [**9**] recently obtained an asymptotic formula for the operators (1). In the present paper we

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