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ON WEAK COMPACTNESS IN L_1 SPACES

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ABSTRACT. We will use the concept of strong generating and a simple renorming theorem to give new proofs to slight generalizations of some results of Argyros and Rosenthal on weakly compact sets in $L_1(\mu)$ spaces for finite measures μ.

1. Introduction. The purpose of this note is to show that a simple transfer renorming theorem explains why $L_1(\mu)$-spaces, for finite measures μ, share some properties with superreflexive spaces, though there is no one-to-one bounded linear operator from $L_1(\mu)$ into any reflexive space if $L_1(\mu)$ is nonseparable [**19**, page 232]. The notations used here are standard (see, e.g., [**11**], where we refer, too, for undefined concepts). By a *measure* we always understand a countably additive measure defined on a σ-algebra Σ of subsets of some nonempty set Ω.

Definition 1. We will say that a Banach space X is *strongly generated by a Banach space* Z if there is a bounded linear operator T from Z into X such that, for every weakly compact set $W \subset X$ and every $\varepsilon > 0$, there exists an $m \in \mathbf{N}$ such that $W \subset mT(B_Z) + \varepsilon B_X$. In this case we will say, too, that Z *strongly generates* X.

Remark 2. Definition 1 is motivated by the concept of a *strongly weakly compactly generated Banach space* (SWCG, for short), introduced by Schlüchtermann and Wheeler [**20**]: A Banach space X is SWCG if there exists a weakly compact subset $K \subset X$ such that, for every weakly compact subset $W \subset X$, we can find an $n \in \mathbf{N}$ such

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