

THE LITTLEWOOD-ORLICZ OPERATOR IDEAL

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ABSTRACT. In this paper, we show that each continuous linear operator from an \mathcal{L}_2 -space to an \mathcal{L}_∞ -space is Littlewood-Orlicz, and each Littlewood-Orlicz operator from a Banach space to an \mathcal{L}_2 -space is 2-summing. As a consequence, Littlewood-Orlicz operators from a Banach space with cotype 2 to an \mathcal{L}_2 -space coincide with 2-summing operators.

1. Introduction. The classes of p -summing operators ($1 \leq p < \infty$) were introduced by Pietsch [16] in 1967. Actually, the class of 1-summing operators was studied before in Grothendieck's Résumé [9] in 1953. Later, Mityagin and Pelczynski [13] and Kwapien [11] studied the classes of (q, p) -summing operators, i.e., operators that take weakly p -summable sequences to absolutely q -summable sequences. Cohen [4] and Apiola [1] studied the classes of 'Cohen's (q, p) -summing operators,' i.e., operators that take weakly p -summable sequences to strongly q -summable sequences. In 1980, Pietsch in his monograph [17] introduced the classes of (r, q, p) -summing operators. In particular, $(1, q, p)$ -summing operators coincide with 'Cohen's (q, p) -summing operators.' Now, with the help of sequential representations of $\ell_p \widehat{\otimes} X$ given by Bu and Diestel [3], $(1, q, p)$ -summing operators from a Banach space X to a Banach space Y are nothing else but operators that take members in $\ell_p \check{\otimes} X$, the injective tensor product of ℓ_p and X , to members in $\ell_q \widehat{\otimes} Y$, the projective tensor product of ℓ_q and Y .

Bu in [2] used $(1, 1, 2)$ -summing operators (called Littlewood-Orlicz operators in [2]) to characterize G.T. spaces with cotype 2. In this paper, we will give some properties of Littlewood-Orlicz operators between \mathcal{L}_p -spaces. The reasons why we are interested in such operators and why we called them Littlewood-Orlicz operators are as follows.

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