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EXISTENCE AND UNIQUENESS RESULTS FOR ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we give some new results concerning solvability of first order singular problems. We study mainly the differential equation $x' = f(t,x)$. We prove that the existence theorem of Caratheodory remains true if f is not defined at the given initial point and satisfies more flexible conditions. This theorem allows us to develop theorems on the existence and uniqueness of the solution of systems of differential equations and high order differential equations. We introduce a more general form of the initial value problems and try to develop this idea.

1. Introduction. In this paper we consider the problem

$$x' = f(t,x), \qquad x(\tau) = \xi, \tag{1}$$

where f is singular in $(t,x) = (\tau,\xi)$. There are different attempts to develop the theory of singular initial value problems. Peano [**9**] and Perron [**10**] have considered the problem (1) when $(\tau,\xi) = (0,0)$, with f continuous, assume the existence of continuous functions $m_1(t)$ and $m_2(t)$ with $m_1(0) = m_2(0)$, $m_1(t) \le m_2(t)$ and

$$D_\pm m_1(t) \le f(t, m_1(t)), \qquad D_\pm m_2(t) \ge f(t, m_2(t)),$$

and prove the existence of a minimal and a maximal solution of (1) between $m_1(t)$ and $m_2(t)$. In 1931, Dragoni [**6**] considered systems and allowed $m_1(0) \le 0 \le m_2(0)$. Recently, Frigon and O'Regan [**7**] have considered noncontinuous $m_1(t)$ and $m_2(t)$. Marcelli and Rubbioni [**8**] have considered the situation where $m_1(t)$ and $m_2(t)$ can cross. Cherpion and De Coster [**2**] have studied the situation where $m_1(t)$ and $m_2(t)$ are not necessarily continuous nor ordered. In most cases, authors tried to develop so-called lower and upper solution methods.

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