ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 39, Number 6, 2009

FINITE RELATIVE DETERMINATION AND THE ARTIN-REES LEMMA

ENRIQUE ANDRADE SOLÍS AND LEÓN KUSHNER

ABSTRACT. If we consider the 4-dimensional vector space of quasi-homogeneous maps in two variables of weights $(\frac{1}{3}, \frac{1}{6})$ and degree 1, we get two 1-parameter families $f_\epsilon(x,y) = x^3 + (\epsilon+1)x^2y^2 + \epsilon xy^4$ and $g_\epsilon(x,y) = x^3 + \epsilon x^2y^2 + xy^4 + \epsilon y^6$. We are interested in comparing the usual finite determination with a suitable group of diffeomorphisms. We prove analogous theorems of finite determination for such group. This work is a continuation of [**3**].

1. Introduction. Following our work [**3**], we consider the quasi-homogeneous maps in two variables of weights $(\frac{1}{3}, \frac{1}{6})$ and degree 1. The group of germs of diffeomorphisms considered there is of the form $h(x,y) = (\alpha x + \beta y^2, \delta y)$. We then study G, a subgroup of the group of diffeomorphisms of the form $h(x,y) = (\alpha x + \beta y^2 + h_1, \delta y + h_2)$ where $h_1 \in m(2)^3$ and $h_2 \in m(2)^2$. We study the concepts of finite determination on the right and finite relative determination for two models, obtaining different numbers. We finish with a version of the Artin-Rees lemma in $\mathcal{E}(n)$, the algebra of smooth germs.

1. Finite determination and finite relative determination. Consider $\mathcal{E}(n)$ the algebra of smooth germs of functions from the n-dimensional Euclidean space to the real numbers.

Theorem 1 (Stefan). *A germ f is k-determined on the right if and only if for each germ $\mu \in m(n)^{k+1}$, we have that*

$$m(n)^{k+1} \subseteq m(n)\left\langle \frac{\partial(f+\mu)}{\partial x_i} \right\rangle + m(n)^{k+2}.$$

2000 AMS *Mathematics subject classification.* Primary 58K40, Secondary 13C05.
Keywords and phrases. Finite determination, finite relative determination, Artin-Rees lemma.
The second author has been partially supported by PAPIIT IN110803-3 "Foliaciones Geométricas y Singularidades." UNAM, México and FAPESP, Brazil.
Received by the editors on April 3, 2006, and accepted on February 22, 2008.
DOI:10.1216/RMJ-2009-39-6-1767