ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 39, Number 4, 2009

CONVERGENCE AND GIBBS PHENOMENON OF PERIODIC WAVELET FRAME SERIES

ZHIHUA ZHANG

ABSTRACT. In this paper, we give integral representations of partial sums of the periodic wavelet frame series and then, based on it, we study convergence and the Gibbs phenomenon of the periodic wavelet frame series.

1. Introduction. It is well known that $\{e^{2\pi int}\}$ is an orthonormal basis for $L^2[0,1]$. The convergence of the Fourier series

$$\sum_n c_n e^{2\pi int}, \qquad c_n = \int_0^1 f(t)e^{-2\pi int}\,dt, \quad f \in L^2[0,1],$$

has been systematically studied [**4, 5**].

If $\{\psi_{m,n}\} = \{2^{m/2}\psi(2^m \cdot -n)\}_{m,n\in Z}$, is an orthonormal basis for $L^2(R)$, then $\{\psi_{m,n}\}$ is called a wavelet basis. From 1986 to present, many wavelet bases have been constructed. The convergence of the wavelet series

$$\sum_{m,n} c_{m,n}\psi_{m,n}(t), \qquad c_{m,n} = \int_R f(t)\overline{\psi}_{m,n}(t)\,dt, \quad f \in L^2(R),$$

was studied deeply [**6, 8, 10**]. Meyer [**7**] first constructed periodic wavelet bases. Skopina [**9**] discussed the convergence of the periodic wavelet series.

Wavelet frames are a generalization of wavelet bases. Recently, periodic wavelet frames were constructed [**11**]. In this paper, we will research convergence and the Gibbs phenomenon of the periodic wavelet frame series.

2000 AMS *Mathematics subject classification.* Primary 42C15.
Keywords and phrases. Convergence, Gibbs phenomenon, periodic wavelet frame series, integral representation.
Received by the editors on September 27, 2006, and in revised form on December 22, 2006.
DOI:10.1216/RMJ-2009-39-4-1373