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THE MAHLER MEASURE OF LINEAR FORMS AS SPECIAL VALUES OF SOLUTIONS OF ALGEBRAIC DIFFERENTIAL EQUATIONS

R. TOLEDANO

ABSTRACT. We prove that for each $n \geq 4$ there is an analytic function $F_n(x)$ satisfying an algebraic differential equation of degree $n+1$ such that the logarithmic Mahler measure of the linear form $\mathbf{L}_n = x_1 + \cdots + x_n$ can be essentially computed as the evaluation of $F_n(z)$ at $z = n^{-1}$. We show that the coefficients of the series representing $F_n(z)$ can be computed recursively using the nth symmetric power of a second order linear algebraic differential equation, and we give an estimate on the growth of these coefficients.

1. Introduction and definitions. Let I denote the unit interval $[0,1]$. Given a Laurent polynomial in several variables $P \in \mathbf{C}[x_1^{\pm}, \dots, x_n^{\pm}]$ the so called *Mahler's measure* of P is defined as

$$M(P) = \mathrm{e}^{m(P)},$$

where

$$m(P) = \int_{I^n} \log |P(\mathrm{e}^{2\pi i x_1}, \dots, \mathrm{e}^{2\pi i x_n})| \, dx_1 \dots dx_n,$$

is the *logarithmic Mahler's measure* of P.

Let us consider the linear form

$$\mathbf{L}_n = x_1 + \cdots + x_n.$$

We will show here that for each $n \geq 4$ there is an analytic function $F_n(z)$ satisfying an algebraic differential equation such that

$$F_n(n^{-1}) = m(\mathbf{L}_n) - \log \sqrt{n} + \gamma/2. \tag{1.1}$$

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