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BOUNDS ON CHARACTERISTIC NUMBERS BY CURVATURE AND RADIUS

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ABSTRACT. We obtain explicit bounds on the Euler characteristic and Pontryagin numbers of closed, connected, oriented Riemannian manifolds in terms of sectional curvature and radius.

1. Introduction. Let M^m be a closed, connected and oriented Riemannian manifold of dimension m with sectional curvature $k \leq \sec \leq K$. By definition, the *radius* of M^m is the number $r = \min_{p \in M^m} \max_{q \in M^m} d(p,q)$, where $d(\ ,\ )$ denotes the distance function on M^m. The aim of this note is to give explicit bounds, in terms of k, K and r, on the Euler-Poincaré characteristic of M^{2n} and the Pontryagin numbers of M^{4n}, when $k \leq 0$. The case $k > 0$ was first studied by Berger, cf. [**1**], considering the diameter instead of the radius. He proved that, if M is a complete Riemannian manifold of dimension $2n$ and $0 < k \leq \sec \leq K$, then

$$|\chi(M)| \leq \frac{K^n}{2^n k^n} \cdot (2n)!,$$

where $\chi(M)$ denotes the Euler-Poincaré characteristic of M. Tsagas obtained explicit bounds for the Pontryagin numbers when $k > 0$, cf. [**12**]. In these cases, since $0 < k \leq \sec \leq K$, it follows from Myers' theorem that M has diameter $d \leq \pi/\sqrt{k}$ (so M must be compact) and the bound on the Euler-Poincaré characteristic will only depend on the sectional curvature bounds. Bishop and Goldberg noted in [**2**] that, using what is now known as Bishop's theorem, it is possible to bound the Euler-Poincaré characteristic of an even dimensional compact Riemannian manifold with bounded sectional curvature and diameter d, generalizing Berger's result. However, they did not carry

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