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FOCK SPACE TECHNIQUES IN TENSOR ALGEBRAS OF DIRECTED GRAPHS

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ABSTRACT. In [**16**], Muhly and Solel developed a theory of tensor algebras over C^*-correspondences that extends the model theory of contractions in $B(H)$. The main examples are generated by Fock spaces, directed graphs and analytic cross products. In this paper we show that many results of tensor algebras of directed graphs, including dilations and commutant lifting theorems for $C_{\cdot 0}$ completely contractive representations, can be deduced from results on Fock spaces. One of the main tools we use is that of Poisson kernels, which we define for arbitrary C^*-correspondences. The Fock space approach allows us to consider "weighted" graphs, where the dilation and commutant lifting theorems hold. Additionally, we prove a rigidity result for submodules of induced representations of directed graphs, and we obtain projective resolutions of graph deformations.

1. Introduction. In the last 30 years there have been many attempts to generalize model theory for contractions in $B(H)$, particularly the Nagy-Foias dilation theory and the commutant lifting theorem. For example, Douglas and Paulsen [**7**] proposed Hilbert module language to extend these results to multivariate function theory. Popescu [**21, 22**] extended them to a noncommutative multivariate setting. And Muhly and Solel [**16, 17**] extended them to tensor algebras over C^*-correspondences (Hilbert bimodules over a C^*-algebra A). The language of Muhly and Solel is very general and it includes as special cases all of the previous examples. Additionally, it includes the tensor algebras generated by directed graphs, analytic cross products, and others.

Many aspects of the Nagy-Foias theory are reduced to study of the unilateral shift in the Hardy space H^2, which has orthonormal basis $\{z^n : n \geq 0\}$. Likewise, many aspects of Popescu's noncommutative

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