

**EXISTENCE OF COMPLEMENTED SUBSPACES
ISOMORPHIC TO l^q IN
QUASI BANACH INTERPOLATION SPACES**

J.A. LÓPEZ MOLINA

ABSTRACT. Let (E_0, E_1) be a compatible couple of quasi Banach spaces and $0 < \theta < 1$, $0 < q < \infty$. We present a sufficient condition in order that the quasi Banach interpolation space $(E_0, E_1)_{\theta, q}$ has complemented subspaces isomorphic to l^q , extending in this way Levy's theorem. As an application we show that every space $(l^{p_0}(\mu), l^{p_1}(\mu))_{\theta, q}$, $0 < p_0 < p_1 \leq \infty$, has complemented subspaces isomorphic to l^q except in the case that $0 < p_\theta < q < 1$, $1/(p_\theta) := (1 - \theta)/p_0 + \theta/p_1$ and $0 < \alpha \leq \mu(\{n\}) \leq \beta < \infty$, $n \in \mathbf{N}$ for some α, β in \mathbf{R} .

1. Introduction. Let (E_0, E_1) be a compatible couple of Banach spaces. Levy has proved in [10] that every interpolated space $(E_0, E_1)_{\theta, q}$, $1 \leq q < \infty$, $0 < \theta < 1$ (real interpolation method) such that $E_0 \cap E_1$ is not closed in $E_0 + E_1$ has complemented subspaces isomorphic to l^q . In [4], Brudnyi and Krugljak extend the scope of Levy's result to more general interpolation functors.

The theory of real interpolation of Banach spaces has been extended to quasi Banach spaces by several authors, see for example, Krée [9], Holmstedt [6], Peetre [15] and Sagher [16]. Essentially with the same hypothesis of Levy, although with a slightly different presentation, we have shown in [12] that such interpolation spaces $(E_0, E_1)_{\theta, q}$, $0 < q < \infty$ of quasi Banach spaces E_0, E_1 have subspaces isomorphic to l^q . In this paper we prove that, *under suitable hypotheses*, these spaces have indeed *complemented subspaces isomorphic to l^q* too.

In general, our notation is standard. We recall that a quasi Banach space is a vector space E over the field \mathbf{K} of real or complex numbers which is complete under the metric $d(x, y) = \|x - y\|$ where $\|\cdot\| : E \rightarrow [0, \infty[$ is a quasi-norm, i.e., a function with properties

2000 AMS *Mathematics subject classification.* Primary 46A45, 46E30, 46M35.

Keywords and phrases. Real interpolation method, quasi Banach spaces.

Partially supported by the MEC and FEDER project MTM2004-02262, net MTM2004-21420-E and AVCIT group 03/050.

Received by the editors on July 27, 2006, and in revised form on October 13, 2006.

DOI:10.1216/RMJ-2009-39-3-899 Copyright ©2009 Rocky Mountain Mathematics Consortium