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COMPARISON OF CUSP FORMS ON $GL(3,\mathbf{Z})$

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ABSTRACT. We give an estimate for the number of Fourier coefficients needed to determine uniquely a cusp form on $GL(3,\mathbf{Z})\backslash PGL(3,\mathbf{R})/O(3)$. This leads to an estimate on the multiplicity of the space of eigenfunctions with fixed infinity type. More precisely we show the multiplicity of the space of eigenfunctions with fixed infinity type, $M(\lambda)=O(\lambda^2)$ where λ is the eigenvalue of the Laplacian.

1. Introduction. Since the early work of Jacobi on θ functions to study the representation of integers as sums of squares, automorphic forms in various guises have played an important role in number theory. Notable among these is Ramanujan's discriminant function whose reciprocal is intimately connected with the study of the partition function in number theory. The automorphic properties of this function, which include periodicity with period one, give rise to a Fourier expansion whose study has been a central theme in number theory for a century. The estimate of the size of the Fourier coefficients is a key element in these applications. Hecke formalized the concept of automorphic forms in the 1920's. He assumes an analytic function on the upper half plane that has only polynomial growth at infinity, and with respect to some discrete subgroup Γ of $SL(2,\mathbf{R})$, the group of symmetries of the upper half plane satisfies

$$f(\gamma z)=(cz+d)^k f(z)$$

where γ, an element of Γ, equals $\left(\begin{smallmatrix} a & b \\ c & d\end{smallmatrix}\right)$. In particular, we have that f is periodic with respect to x, so we have a Fourier expansion.

In the 1940's, Maass generalized this notion by considering functions that are not holomorphic but are eigenfunctions of the Laplace-Beltrami operators. These functions, in particular when Γ is $SL(2,\mathbf{Z})$, have been of tremendous usefulness in the study of the Riemann zeta function, and in various aspects of the study of Kloosterman sums.

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