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CONSTRUCTION OF BIHOLOMORPHIC CONVEX MAPPINGS ON D_p IN C^n

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ABSTRACT. In this paper, we first prove some sufficient conditions for the biholomorphic convex mappings on the Reinhardt domain D_p $(p_j \geq 2,\ j = 1, \dots, n)$ in C^n. From these, we construct some concrete examples of biholomorphic convex mappings on the Reinhardt domain D_p $(p_j \geq 2,\ j = 1, \dots, n)$. We also introduce a linear operator and a subclass of biholomorphic convex mappings for the purpose of constructing some biholomorphic convex mappings on D_p in C^n.

Let C^n be the vector space of n-complex variables $z = (z_1, z_2, \dots, z_n)$ with the usual inner product $\langle z, w \rangle = \sum_{j=1}^n z_j \overline{w}_j$, where $w = (w_1, w_2, \dots, w_n) \in C^n$. Suppose that D is a domain in C^n. If, for every $z \in D$, $\lambda \in C$ and $|\lambda| \leq 1$, we have $\lambda z \in D$, then we call D a balanced domain. The *Minkowski functional* of a balanced domain D is defined by

$$\rho(z) = \inf\left\{t > 0, \frac{z}{t} \in D\right\}, \quad z \in C^n.$$

Suppose that D is a bounded convex balanced domain in C^n and $\rho(z)$ is the Minkowski functional of D. Then $\rho(\bullet)$ is a norm of C^n, and $D = \{z \in C^n : \rho(z) < 1\}$, $\rho(\lambda z) = |\lambda|\rho(z)$, where $\lambda \in C$, $z \in C^n$ and $\rho(z) = 0$ if and only if $z = 0$, see [**15**].

Assume $p_j > 1$, $j = 1, 2, \dots, n$. Let $D_p = \{(z_1, z_2, \dots, z_n) \in C^n : \sum_{j=1}^n |z_j|^{p_j} < 1\}$. Then D_p is a bounded convex balanced domain in

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