PERIODIC BOUNDARY VALUE PROBLEMS
FOR SECOND ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT. This paper presents sufficient conditions for the existence of solutions to periodic boundary-value problems of second order differential equations. Our results are obtained via a new fixed point theorem which is developed in the paper and lead to new existence principles.

1. Introduction. Let $J = [0, T]$ be a compact interval, and let $\|x\| = \max\{|x(t)| : t \in J\}$, $\|x\|_L = \int_0^T |x(t)| \, dt$ and $\|(x, a, b)\|_0 = \|x\| + |a| + |b|$ be the norms in Banach spaces $C(J), L_1(J)$ and $C(J) \times \mathbf{R}^2$, respectively.

This paper is concerned with the existence of solutions for the periodic boundary value problem of second order differential equations (PBVP)

$$(1) \quad \begin{cases} (x'(t) + g(t, x(t), x'(t)))' = f(t, x(t), x'(t)), \\ x(0) = x(T), \quad x'(0) = x'(T), \end{cases}$$

and the boundary value problem (BVP)

$$(2) \quad \begin{cases} (x'(t) + g(t, x(t), x'(t)))' = f(t, x(t), x'(t)), \\ x(0) = A, x(T) = B, \end{cases}$$

where $g \in C(J \times \mathbf{R}^2)$, $g(0, x, y) = g(T, x, y)$ for $(x, y) \in \mathbf{R}^2$ and f satisfies the local Carathéodory conditions on $J \times \mathbf{R}^2$.

The solvability of periodic problems and boundary value problems (1), (2) have been widely investigated with $g = 0$ and one-sided growth restrictions to the third argument in the nonlinearity f. We refer, for instance, to [**3, 6, 7, 9**] and references therein. Reference [**8**] has considered periodic boundary value problems (1) and has shown that the existence of solutions for PBVP(1) is equivalent to the existence