

APPROXIMATION FOR MODIFIED BASKAKOV DURRMEYER TYPE OPERATORS

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ABSTRACT. In the present paper, we study a certain integral modification of the well-known Baskakov operators with the weight function of Beta basis function. We establish some local and global direct results in ordinary and simultaneous approximation for these new operators.

1. Introduction. For $x \in [0, \infty)$ and $\alpha > 0$, we consider a certain type of Baskakov-Durrmeyer operators as

(1.1)

$$\begin{aligned} B_{n,\alpha}(f(t), x) &= \sum_{k=1}^{\infty} p_{n,k,\alpha}(x) \int_0^{\infty} b_{n,k,\alpha}(t) f(t) dt + (1 + \alpha x)^{-n/\alpha} f(0) \\ &= \int_0^{\infty} W_{n,\alpha}(x, t) f(t) dt \end{aligned}$$

where

$$\begin{aligned} p_{n,k,\alpha}(x) &= \frac{\Gamma(n/\alpha + k)}{\Gamma(k + 1)\Gamma(n/\alpha)} \cdot \frac{(\alpha x)^k}{(1 + \alpha x)^{(n/\alpha)+k}} \\ b_{n,k,\alpha}(t) &= \frac{\alpha\Gamma(n/\alpha + k + 1)}{\Gamma(k)\Gamma(n/\alpha + 1)} \cdot \frac{(\alpha t)^{k-1}}{(1 + \alpha t)^{(n/\alpha)+k+1}} \end{aligned}$$

and

$$W_{n,\alpha}(x, t) = \sum_{k=1}^{\infty} p_{n,k,\alpha}(x) b_{n,k,\alpha}(t) + (1 + \alpha x)^{-n/\alpha} \delta(t),$$

$\delta(t)$ being the Dirac delta function.

The operators defined by (1.1) are the integral modification of the well-known Baskakov operators having weight functions of some beta

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