ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 39, Number 2, 2009

SUFFICIENT CONDITIONS FOR FACTORABLE MATRICES TO BE BOUNDED OPERATORS ON $\mathcal{A}_k$

E. SAVAŞ, H. ŞEVLI AND B.E. RHOADES

ABSTRACT. A factorable matrix A is a lower triangular matrix with entries $a_{nk} = a_n b_k$. The sequence space $\mathcal{A}_k$ is defined in (2). In this paper we determine sufficient conditions for a nonnegative factorable matrix A to be a bounded operator on $\mathcal{A}_k$, i.e., $A \in B(\mathcal{A}_k)$. As corollaries we obtain sufficient conditions for the discrete Cesáro, terraced, and p-Cesáro matrices defined by Rhaly, to be in $B(\mathcal{A}_k)$.

1. Introduction. Let T be an infinite matrix, $\{s_n\}$ a sequence. Then the nth term, t_n, of the T transform of $\{s_n\}$ is given by

$$t_n = \sum_{v=0}^{\infty} t_{nv} s_v.$$

A series $\sum x_n$, with partial sums s_n, is said to be k-absolutely summable by T for $k \geq 1$, written $\sum x_n$ is $|T|_k$, if

$$\sum_{n=1}^{\infty} n^{k-1} |\Delta t_{n-1}|^k < \infty, \tag{1}$$

where Δ is the forward difference operator defined by $\Delta t_{n-1} = t_{n-1} - t_n$.

Let (C, α) denote the Cesáro matrix of order $\alpha > -1$, σ_n^α its nth transform of a sequence $\{s_n\}$. Using (1) with $t_n = \sigma_n^\alpha$, Flett [**3**] proved that, if a series $\sum x_n$ is summable $|C, \alpha|_k$ then it is summable $|C, \beta|_r$ for $\alpha > -1$, $r \geq k \geq 1$, $\beta > \alpha + 1/k - 1/r$. Setting $r = k$ gives an inclusion type theorem for Cesáro matrices.

2000 AMS *Mathematics subject classification.* Primary 40D15.
Keywords and phrases. Bounded operator, factorable matrices, Rhaly matrices, $\mathcal{A}_k$ spaces.
Received by the editors on July 12, 2006.

DOI:10.1216/RMJ-2009-39-2-633