

WAVELET TRANSFORM ON SPACES OF TYPE W

R.S. PATHAK AND GIREESH PANDEY

ABSTRACT. The continuous wavelet transform is studied on certain Gelfand Shilov spaces of type W . The continuity and boundedness results for continuous wavelet transform are obtained on some suitably designed spaces of type W defined on $\mathbf{R} \times \mathbf{R}_+$, $\mathbf{C} \times \mathbf{R}_+$ and $\mathbf{C} \times \mathbf{C}$.

1. Introduction. The spaces of W -type were studied by Gelfand and Shilov [2]. They investigated the behavior of Fourier transformation on W -spaces. Also, W -spaces are applied to the theory of partial differential equations.

Pathak [4] and van Eijndhoven and Kerkhof [1] introduced new spaces of W -type and investigated the behavior of Hankel transformation over them.

The wavelet transform on Schwartz space $\mathcal{S}(\mathbf{R})$ and spaces of Sobolev type have been studied by many authors, see for example Holschneider [3]. In this paper, motivated by the work of Pathak [5] and Pathak and Upadhyay [6] we recall characterizations of W -type spaces introduced in [2] and study the behavior of continuous wavelet transform over them.

The continuous wavelet transform of a function ϕ with respect to the wavelet ψ is defined by

$$(1.1) \quad (\mathcal{W}_\psi \phi)(\sigma, a) = \tilde{\Phi}(\sigma, a) = \int_{-\infty}^{\infty} \phi(t) \overline{\psi\left(\frac{t-\sigma}{a}\right)} \frac{dt}{a},$$

provided the integral exists, where $a \in \mathbf{R}_+$ and $\sigma \in \mathbf{R}$. If $\phi \in L^2(\mathbf{R})$ and $\psi \in L^2(\mathbf{R})$, then using the Parseval formula for Fourier transform,

2000 AMS *Mathematics subject classification.* Primary 42C40, 46F12, 46F15.

Keywords and phrases. W -type spaces, continuous wavelet transform.

The work of the second author was supported by CSIR (New Delhi), Grant No. 9/13(04)/2003/ EMR-I.

Received by the editors on June 13, 2006, and in revised form on September 7, 2006.

DOI:10.1216/RMJ-2009-39-2-619 Copyright ©2009 Rocky Mountain Mathematics Consortium