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PRIMES BETWEEN CONSECUTIVE POWERS

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ABSTRACT. A well-known conjecture about the distribution of primes asserts that between two consecutive squares there is always at least one prime number. The proof of this conjecture is quite out of reach at present, even under the assumption of the Riemann Hypothesis. This paper is concerned with the distribution of prime numbers between two consecutive powers of integers, as a natural generalization of the aforementioned conjecture. The results follow from the proprieties of the exceptional set for the distribution of prime in short intervals.

1. Introduction. A well-known conjecture about the distribution of primes asserts that all intervals of type $[n^2, (n+1)^2]$ contain at least one prime.

The proof of this conjecture is quite out of reach at present, even under the assumption of the Riemann hypothesis. Goldston proved the conjecture assuming a strong form of the Montgomery conjecture, see [**3**]. The author improved this result proving that all intervals of type $[n^2, (n+1)^2]$ contain the expected number of primes, for $n \to \infty$, assuming a weaker hypothesis about the behavior of Selberg's integral in short intervals, see Bazzanella [**1**].

This paper is concerned with the distribution of prime numbers between two consecutive powers of integers, as a natural generalization of the above conjecture.

The well-known result of Huxley about the distribution of primes in short intervals, see [**5**], implies that the intervals $[n^\alpha, (n+1)^\alpha]$ contain the expected number of primes for $\alpha > (12/5)$ and $n \to \infty$.

As was observed, we are unable to prove a similar result for $\alpha = 2$, even under the assumption of the Riemann hypothesis. However, we can obtain some results under the assumption of the Lindelöf

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