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COMPACT ENDOMORPHISMS OF INFINITELY DIFFERENTIABLE LIPSCHITZ ALGEBRAS

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ABSTRACT. Let X be a perfect compact plane set and $0 < \alpha \leq 1$. The Lipschitz algebra of order α, $\mathrm{Lip}\,(X, \alpha)$ is the algebra of all complex-valued functions f on X for which

$$p_\alpha(f) = \sup \left\{ \frac{|f(z) - f(w)|}{|z - w|^\alpha} : z, w \in X, z \neq w \right\} < \infty.$$

Denote by $\mathrm{Lip}^{\,\infty}(X, \alpha)$ the algebra of functions f on X whose derivatives of all orders exist and $f^{(n)} \in \mathrm{Lip}\,(X, \alpha)$ for all n. Let (M_n) be a sequence of positive numbers satisfying $M_0 = 1$ and $M_{n+m}/M_n M_m \geq \binom{n+m}{n}$ for all nonnegative integers m, n, and let

$$\mathrm{Lip}\,(X, M, \alpha) = \left\{ f \in \mathrm{Lip}^{\,\infty}(X, \alpha) : \|f\| = \sum_{k=0}^{\infty} \frac{\|f^{(k)}\|_\alpha}{M_k} < \infty \right\},$$

where $\|f\|_\alpha = \|f\|_X + p_\alpha(f)$. In this paper we study the endomorphisms of this kind of Lipschitz algebra. When $\mathrm{Lip}\,(X, M, \alpha)$ is a natural Banach function algebra, every nonzero endomorphism T of $\mathrm{Lip}\,(X, M, \alpha)$ has the form $Tf = f \circ \varphi$, for some self-map φ of X. First we give some sufficient conditions for φ to induce an endomorphism of $\mathrm{Lip}\,(X, M, \alpha)$. Then we investigate necessary and sufficient conditions for these endomorphisms to be compact. Finally, we determine the spectra of compact endomorphisms of these algebras.

1. Introduction and preliminaries. In this note we investigate endomorphisms of a class of Lipschitz algebras of infinitely differentiable functions. Let X be a perfect compact plane set and $0 < \alpha \leq 1$.

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