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MODULARITY OF SOME NONRIGID DOUBLE OCTIC CALABI-YAU THREEFOLDS

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ABSTRACT. In this paper we discuss four methods of proving modularity of Calabi-Yau threefolds with $h^{12} = 1$: existence of elliptic ruled surfaces inside (Hulek-Verrill), correspondence with a product of an elliptic curve and a K3 surface (Livné-Yui), correspondence with a (modular) rigid Calabi-Yau threefold, and existence of an involution splitting the fourdimensional representation into two-dimensional subrepresentations.

We apply these methods to prove modularity of 17 out of 18 double octic Calabi-Yau threefolds for which "numerical evidence of modularity" was found in the second author's recently published book [**11**].

We observe that modularity holds for those elements in a pencil having certain additional geometric properties. In the proofs we use representations of the considered Calabi-Yau threefolds as a Kummer fibration associated to a fiber product of rational elliptic fibrations.

1. Introduction. The modularity conjecture for Calabi-Yau manifolds predicts that every Calabi-Yau manifold should be modular in the sense that its L-series coincides with the L-series of some automorphic form(s). The case of rigid Calabi-Yau threefolds was (almost) solved by Dieulefait and Manoharmayum in [**6, 7**]. On the other hand, in the nonrigid case it is not even clear which automorphic forms should appear.

Examples of nonrigid modular Calabi-Yau threefolds were constructed by Livné and Yui [**10**], Hulek and Verrill [**8, 9**] and Schütt [**16**]. In these examples modularity means a decomposition of the associated Galois representation into two- and four-dimensional subrepresentations with L-series equal to $L(g_4, s)$, $L(g_2, s-1)$ or $L(g_2 \otimes g_3, s)$, where g_k is a weight k cusp form. The summand with L-series equal to $L(g_2 \otimes g_3, s)$

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