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MAXIMUM LIKELIHOOD ESTIMATION FOR SIMPLEX DISTRIBUTION NONLINEAR MIXED MODELS VIA THE STOCHASTIC APPROXIMATION ALGORITHM

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ABSTRACT. Longitudinal continuous proportional data is common in many fields such as biomedical research, psychological research and so on, e.g., the percent decrease in glomerular filtration rate at different follow-up times from the baseline. As shown in Song and Tan [**16**] such data can be fitted with simplex models. However, the original models of [**16**] for such longitudinal continuous proportional data assumed a fixed effect for every subject. This paper extends the models of Song and Tan [**16**] by adding random effects, and proposes simplex distribution nonlinear mixed models which are one kind of nonlinear reproductive dispersion mixed model. By treating random effects in the models as hypothetical missing data and applying the Metropolis-Hastings (M-H) algorithm, this paper develops the stochastic approximation (SA) algorithm with Markov chain Monte-Carlo (MCMC) method for maximum likelihood estimation in the models. Finally, for ease of comparison, the method is illustrated with the same data from an ophthalmology study on the use of intraocular gas in retinal surgeries in [**16**].

1. Introduction. Dispersion models, which contain a broader class of distributions that accommodate a large number of different data types, were defined in [**9**]. Besides those familiar exponential family distributions, the simplex distribution of Barndorff-Nielsen and Jørgensen [**1**] also represents a special dispersion model for proportional data and is of particular interest in this paper. Based on this distribution, Song and Tan [**16**] developed a marginal simplex model for longitudinal continuous proportional data and assumed a constant dispersion in their model, and this model was used to analyze the eye surgery data in [**14**]; [**15**] further assumed a varying dispersion on the basis of [**16**] and re-analyzed the same surgery data; and Zhang [**18**, page 4] proposed

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