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DYNAMIC BEHAVIOR OF A DELAYED IMPULSIVE SEIRS MODEL IN EPIDEMIOLOGY

TAILEI ZHANG AND ZHIDONG TENG

ABSTRACT. A delayed SEIRS epidemic model with pulse vaccination is investigated. Using Krasnoselskii's fixed-point theorem, the infection-free periodic solution is obtained. Some new threshold values $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{R}_3$ are obtained for dynamic behavior of the solutions. We point out, if $\mathcal{R}_1 < 1$, the infectious population disappear, i.e., the disease dies out, while if $\mathcal{R}_2 > 1$ or $\mathcal{R}_3 > 1$, the infectious permanent, the infectious population will ultimately remain above a positive level. An explicit formula is obtained by which the eventual lower bound of infectious individuals can be computed when $\mathcal{R}_2 > 1$. Our results indicate that a large pulse vaccination rate will have some active effects to prevent or curtail the spread of the disease. Furthermore, we only proved the existence of $\mathcal{R}_3$ based upon some abstract theories.

1. Introduction. The spread of infectious diseases is often described mathematically by compartmental models. A theory of epidemics was derived by Kermack, a chemist, and Mckendrick, a physician, who worked at the Royal College of Surgeons in Edinburgh between 1900 and 1930. They introduced and used many novel mathematical ideas in studies of populations [**6, 13, 17**]. From then on, most of the research literature assumed that the disease latent period is negligible, i.e., once infected, each susceptible individual (in class S) becomes infectious (in class I), instantaneously and later recovers (in class R) with a permanent or temporary acquired immunity. Today, we usually call these compartmental models SIR models or SIRS models with each letter referring to a 'compartment' in which an individual can reside. The SIR and SIRS models have been studied in much liter-

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The first author is the corresponding author.

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