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NECESSARY AND SUFFICIENT CONDITIONS FOR GLOBAL ATTRACTIVITY OF HOPFIELD-TYPE NEURAL NETWORKS WITH TIME DELAYS

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ABSTRACT. In this paper, two classes of two- and three-dimensional Hopfield-type neural networks with time delays are considered by using a completely different method from known results. Some necessary and sharper sufficient criteria for the global attractivity of equilibria of the neural networks are presented.

1. Introduction. It is well known that early study on stability of Hopfield-type neural networks mainly dealt with ordinary differential equation models (see, for example, [**11, 12**]) in which the updating and propagation are assumed to occur *instantaneously*. However, strictly speaking, the integration and communication delays are ubiquitous both in biological and in artificial neural networks. Hence, investigation on the stability of neural networks with time delays has attracted considerable interest in recent years (see, for example, [**1–9, 19, 20**] and [**14–18, 21–25**]).

Hopfield-type neural networks have a broad spectrum of application in various optimization, associative memories, and engineering problems (see, for example, [**11, 12**]). As is known, engineering applications of neural networks, such as optimization and association, rely crucially on the dynamical behaviors of neural networks. Therefore, qualitative analysis of neurodynamics is indispensable for the practical design of neural network models and tools. When neural networks are applied as associative memories, the equilibria represent the stored patterns, and their stability means that the stored patterns can be retrieved even in the presence of noise. When applied as optimization solvers, the equilibria of the networks characterize all possible optimal solutions of the

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