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A CLASS OF NONLINEAR MULTISTAGE DYNAMICAL SYSTEM AND ITS OPTIMAL CONTROL

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ABSTRACT. In this paper we study a nonlinear multistage dynamical system as well as its optimal control. Specifically, based on the batch fermentation including three different phases of bio-dissimilation of glycerol to 1,3-Propanediol by *Klebsiella pneumonicae*, the nonlinear multi-stage dynamical system is proposed. Then we discuss several properties of this nonlinear system. In order to optimize the initial state such that the concentration of 1,3-Propanediol at terminal time is as large as possible, an optimal control model is established. We investigate the existence of the local maximizer. Furthermore, by using the infinite-dimensional optimization principle, the necessary condition for the optimal control problem is obtained. Finally, employing some properties of the feasible region, infinite-dimensional constraints can be transformed into finite-dimensional constraints.

1. Introduction. In nature, kinestate in many problems, such as the control of modifying 3D horizontal wells trajectory while drilling [**9**], biotechnology, macroscopical or microcosmic control of economy and so on, has some characteristics, such as jump or speed change, for example. For this kinestate the common continuous differential dynamical system is not so valid that a new dynamical system, the nonlinear multistage dynamical system, has to be adopted. In this paper, we investigate both properties and optimal control of a nonlinear multistage dynamical system developed from a practical problem. Specifically, the system is developed from the batch fermentation of the bio-dissimilation of glycerol to 1,3-Propanediol by *Klebsiella pneumonicae* which is a popular subject.

Keywords and phrases. Bio-dissimilation, nonlinear multistage dynamical system, batch fermentation, optimal control.

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