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A STAGE STRUCTURED PREDATOR-PREY MODEL WITH TIME DELAYS

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ABSTRACT. In this paper, a stage structured predator-prey system with two time delays (maturation delay and gestation delay) is studied. Qualitative analysis of the model such as the stability of equilibria and Hopf bifurcation is provided. By using an iterative technique and comparison arguments, sufficient conditions are derived for the global asymptotical stability of the positive equilibrium and the boundary equilibrium of the model. The special case where one of the delays can be ignored is considered, and sufficient conditions are obtained for the occurrence of a Hopf bifurcation of periodic solutions when the gestation delay is varied. It is also shown that varying the maturation delay does not affect the stability of the positive equilibrium.

1. Introduction. This paper presents the model development and analysis of predation involving species with stage structures and gestation delays. This study is motivated by the distinct characteristics at different stages of growth and development of the prey population. Such differences induced by stage structures of the prey species should not be ignored since, for example, the immature individuals cannot reproduce, while the mature may have greater survival capability in addition to their fecundity.

Stage-structured predator-prey models have received substantial in the literature [**1, 2, 4–14**]. Aiello and Freedman [**1**] proposed and studied the following now well-known single species model with structured delay

$$\begin{aligned}\dot{x}_{im}(t) &= \alpha x_m(t) - \gamma x_{im}(t) - \alpha e^{-\gamma\tau} x_m(t-\tau),\\ \dot{x}_m(t) &= \alpha e^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t),\end{aligned}$$

where $x_{im}(t)$ and $x_m(t)$ are densities of the immature and the mature populations, respectively. This model has then been extended to cover

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