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GLOBAL STABILITY AND HOPF BIFURCATION ON A PREDATOR-PREY SYSTEM WITH DIFFUSION AND DELAYS

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ABSTRACT. In this paper a predator-prey system with diffusion and two delays is considered, where the time delays are regarded as parameters. Its dynamics are studied in terms of permanence analysis and Hopf bifurcation analysis. By constructing a suitable Lyapunov function, sufficient conditions are obtained for both local and global stability of the positive equilibrium. An example is presented to show the main conclusion.

1. Introduction. In this paper we consider a system composed of two patches. The system has the predator species and the prey species. The prey species can diffuse between two patches, and the predator species is confined to one of the patches. Several authors established the persistence for predator-prey system with diffusion [**1, 2, 5–7, 9, 10, 13**].

Now we consider the following predator-prey system with diffusion and two discrete delays

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\Big(r_1 - a_1x_1(t) - \frac{c_1y(t-\tau_1)}{1+kx_1(t)}\Big) + \delta(x_2(t)-x_1(t)),\\ \dot{x}_2(t) &= x_2(t)\Big(r_2 - a_2x_2(t-\tau_2)\Big) + \delta(x_1(t)-x_2(t)), \qquad (1.1)\\ \dot{y}(t) &= y(t)\Big(-d_1 + \frac{c_2x_1(t)}{1+kx_1(t)} - d_2y(t)\Big), \end{aligned}$$

where $x_1(t)$ and $y(t)$ are the numbers of prey and predator species in patch 1; $x_2(t)$ is the number of prey species in patch 2. The term

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