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DYNAMICAL BEHAVIOR OF AN EPIDEMIC MODEL WITH COINFECTION OF TWO DISEASES

JIANQUAN LI, LADI WANG, HUIWEN ZHAO AND ZHIEN MA

ABSTRACT. We have formulated a simple epidemiological model with two diseases that can coinfect a single host. The first disease is assumed to be chronic, the second one acute. For infectives infected only by the first disease, we introduce the age of infection. For these two diseases, we obtain their reproduction numbers, respectively, and establish conditions for the existence and stability of the disease-free equilibrium, the boundary equilibrium, and the positive (coexistent) equilibrium. For infectious individuals infected only by the first disease, when some transfer rates depend on the age of infection and the corresponding model is governed by partial differential equations (PDEs), we give a sufficient condition for the existence of positive equilibrium, and its stability is determined by a transcendental equation; when all the associated rates are independent of the age of infection, the corresponding models are ordinary differential equations (ODEs). We obtain complete results on dynamics, find that the coexistent equilibrium of two diseases is globally stable if they exist, and find that the boundary equilibrium is globally stable if it is locally stable. Finally, we find that there is a difference between PDE and ODE models.

1. Introduction. It is well known that a carrier of human immunodeficiency virus (HIV) or a patient with tuberculosis (TB) commonly suffers with a slow progressing disease which lasts for a few years [**4, 5, 6, 9, 17, 20**], i.e., these diseases are chronic. Generally, infecting ability of a patient with the chronic disease may vary with the change of the age of infection (the time lapsed since infection) [**9, 13, 18**]. Thus, to formulate the spread of infection, it is necessary to incorporate the age of infection into an epidemic model with a chronic disease. On the other hand, a patient infected by a chronic disease may also be infected by other acute diseases [**1, 8**] such that coinfection of

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