## ANALYSIS OF AN SEIRS EPIDEMIC MODEL WITH TIME DELAYS AND PULSE VACCINATION

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ABSTRACT. Pulse vaccination is an important strategy for the elimination of infectious diseases. An SEIRS epidemic model with time delays and pulse vaccination is formulated in this paper. By the comparison theorem in impulsive differential equations, we obtain that the infection-free periodic solution is globally attractive if the pulse vaccination rate is larger than  $\theta^*$ . Moreover, we show that the disease is uniformly persistent if the pulse vaccination rate is less than  $\theta_*$  under appropriate conditions. The permanence of the model is investigated analytically.

1. Introduction. Recently epidemic models have been studied by many authors, especially the models with time delays. Cooke and Driessche in literature [4] investigated an SEIRS model with two time delays,

(1.1) 
$$\begin{cases} dS(t)/dt = bN(t) - bS(t) - (\lambda S(t)I(t)/N(t)) \\ + \gamma I(t - \tau)e^{-(b+d)\tau}, \\ E(t) = \int_{t-\omega}^{t} (\lambda S(u)I(u)/N(u))e^{-b(t-u)} du, \\ dI(t)/dt = (\lambda S(t - \omega)I(t - \omega)/N(t - \omega))e^{-b\omega} \\ - (b + \gamma + d)I(t), \\ R(t) = \int_{t-\tau}^{t} \gamma I(u)e^{-(b+d)(t-u)} du, \end{cases}$$

where S(t), E(t), I(t) and R(t) denote the population size that are susceptible, exposed, infectious and recovered at time t, respectively, b is the natural birth and death rate of the population, d is the disease-related death rate,  $\lambda$  is the average number of adequate contacts of an

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