ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 38, Number 5, 2008

STABILITY ANALYSIS OF THE NONLINEAR IMPULSIVE SYSTEM IN MICROBIAL FED-BATCH FERMENTATION

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ABSTRACT. In this study, the nonlinear impulsive dynamical system of fed-batch fermentation is investigated in the process of bio-dissimilation of glycerol to 1,3-propanediol. Several stability criteria are established by employing the method of Lyapunov functions. The ranges of parameters' value in the system are obtained according to the actual microbial fermentation.

1. Introduction. 1,3-propanediol(1,3-PD) possesses potential applications on a large commercial scale, especially as a monomer of polyesters or polyurethanes. Its microbial production has recently been paid worldwide attention for its low cost, high production and no pollution, etc. [**1**]. Among all kinds of microbial production of 1,3-PD, dissimilation of glycerol to 1,3-PD by *Klebsiella pneumoniae* has been widely investigated since the 1980s due to its high productivity [**5, 6**]. Experimental investigations showed that the fermentation of glycerol by *K. pneumoniae* is a complex bioprocess, since the microbial growth is subjected to multiple inhibitions of substrate and products. Research concerning fermentation includes the quantitative description of cell growth kinetics of multiple-inhibitions, metabolic overflow kinetics of substrate consumption and product formation in continuous cultures, feeding strategy of glycerol in fed-batch culture, and so on [**4**]. In this research on fed-batch culture, all numerical results are based on continuous dynamical models, and big errors exist between computational and experimental results. In fact, impulsive phenomena exist in fed-batch culture, so the process characterized by continuous models is not fit for the actual process any longer. In order to characterize the actual process, impulsive differential equations are applied to the fed-batch fermentation [**2**]. Parameters in the continuous system are not fit for

2000 AMS *Mathematics subject classification.* Primary 34K45, 49N25, 49N90.
Keywords and phrases. Stability, Lyapunov function, nonlinear impulsive system.
Received by the editors on August 19, 2007.
DOI:10.1216/RMJ-2008-38-5-1377