ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 38, Number 5, 2008

GLOBAL BEHAVIOR OF A REACTION-DIFFUSION VOLTERRA EQUATION WITH VARIABLE COEFFICIENTS

YONG-HONG FAN AND LIN-LIN WANG

ABSTRACT. By using the method of sub- and supersolutions, the technique of monotone iteration and the Lyapunov functional method, we investigated the permanent behavior and global stability of a reaction-diffusion Volterra equation with variable and constant coefficients.

1. Introduction. In [**7**] Volterra proposed a simple model to describe the evolution of a single species population which has the form

$$x'(t) = x(t)\left(a - bx(t) - \int_0^t H(t-s)x(s)\,ds\right), \quad t \geq 0. \tag{1.1}$$

This model describes the growth of a single species whose population density at time t is $x(t)$. Here a and b are positive constants, the term $x(t)(a - bx(t))$ stands for logistic growth and $x(t)\int_0^t H(t-s)x(s)\,ds$ means a hereditary effect, representing competition for resources, which depends on the population's history.

It is worth considering equation (1.1) with diffusion. We assume that the population lives in a bounded domain $\Omega \subset R^n$ and that there is no migration of individuals across the boundary $\partial\Omega$; we further assume that $\partial\Omega$ is a C^2-manifold.

Define the Laplace transformation $\widehat{f}$ of f by

$$\widehat{f}(t) = \int_0^\infty e^{-ts} f(s)\,ds.$$

Keywords and phrases. Reaction-diffusion Volterra equation, sub- and super-solutions, Lyapunov functional method, global stability.
Supported by the Natural Science Foundation of Ludong University (24070301, 24070302, 24200301), Program for Innovative Research Team in Ludong University, and China Postdoctoral Science Foundation funded project (20080430314).
The second author is the corresponding author.
Received by the editors on June 4, 2007, and in revised form on November 27, 2007.

DOI:10.1216/RMJ-2008-38-5-1335