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PERTURBATION RESULTS FOR LINEAR OPERATORS AND APPLICATION TO THE TRANSPORT EQUATION

KHALID LATRACH AND J. MARTIN PAOLI

ABSTRACT. We prove that the components of the Fredholm domains of closed linear operators on Banach spaces remain invariant under additive perturbations belonging to broad classes of perturbing operators. Although our approach is somewhat different than the standard one used to discuss the stability of essential spectra of such operators, our results provide a natural extension of many known ones in the literature and, in particular, of those obtained in the works [**18, 19**]. Of particular interest is the case of polynomially compact operators which furnishes the convenient setting to describe the essential spectra of multi-dimensional neutron transport operators on L_1 spaces which is the topic of the last section.

1. Introduction. Let X and Y be two infinite dimensional complex Banach spaces, and let $\mathcal{C}(X,Y)$, respectively $\mathcal{L}(X,Y)$, denote the set of all closed, densely defined, respectively bounded, linear operators from X into Y. The subset of all compact, respectively finite rank, operators of $\mathcal{L}(X,Y)$ is designated by $\mathcal{K}(X,Y)$, respectively $\mathcal{F}_0(X,Y)$. If $A \in \mathcal{C}(X,Y)$, we write $N(A) \subseteq X$ and $R(A) \subseteq Y$ for the null space and range of A. We set $\alpha := \dim N(A)$, $\beta := \operatorname{codim} R(A)$. The set of upper semi-Fredholm operators is defined by

$$\Phi_+(X,Y) = \{A \in \mathcal{C}(X,Y) : \alpha(A) < \infty \text{ and } R(A) \text{ is closed is in } Y\},$$

and the set of lower semi-Fredholm operators is defined by

$$\Phi_-(X,Y) = \{A \in \mathcal{C}(X,Y) : \beta(A) < \infty \quad (\text{and } R(A) \text{ is closed in } Y)\}.$$

Operators in $\Phi_\pm(X,Y) := \Phi_+(X,Y) \cup \Phi_-(X,Y)$ are called semi-Fredholm operators from X on Y while $\Phi(X,Y) = \Phi_+(X,Y) \cap \Phi_-(X,Y)$ denotes the set of Fredholm operators from X on Y. For

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